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NUMERICAL MODELING OF THE SEISMIC RESPONSE OF BUILDINGS WITH ENERGY DISSIPATORS

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Abstract. *The poor performance of many framed RC structures in recent strong earthquakes has alerted about the need of improving their seismic behavior especially when they are designed according to obsolete seismic codes. Sometimes, RC buildings show a low level of structural damping, important second order effects and low ductility of the connecting joints, among other defects. These characteristics allow proposing the use of energy dissipating devices for improving their seismic behavior, controlling their lateral displacements, providing additional damping and ductility. In this work, the nonlinear dynamic response of RC buildings with energy dissipating devices is studied using advanced computational techniques. A fully geometric and constitutive nonlinear model for the description of the dynamic behavior of framed structures is used. The model proposed for the structures and the dissipating devices is based on the geometrically exact formulation for beams which considers finite deformation and finite strains. The equations of motion of the system are expressed in terms of sectional forces and generalized strains and the dynamic problem is solved using the displacement based method formulated in the finite element framework. An appropriated version of Newmark's integration scheme is used in updating the kinematics variables in a classical Newton type iterative scheme. Each material point of the cross section is assumed to be composed of several simple materials with their own constitutive laws developed in terms of the material description of the First Piola Kirchhoff stress vector. Appropriated constitutive laws for concrete and for steel reinforcements are provided. The simple mixing theory is used to treat the resulting composite. A specific finite element based on the beam theory is proposed for modeling the energy dissipating devices. Several constitutive descriptions in terms of force and displacements are provided for the dissipators. Special attention is paid to the development of local and global damage indices capable of describing the residual strength of the buildings. Finally, several numerical tests are carried out to validate the ability of the model to reproduce the nonlinear seismic response of RC buildings with energy dissipating devices.*

1 INTRODUCTION

Conventional seismic design practice permits designing reinforced concrete (RC) structures for forces lower than those expected from the elastic response on the premise that the structural design assures significant structural ductility [6]. Frequently, the dissipative zones are located near the beam-column joints and, due to cyclic inelastic incursions during earthquakes, several structural members can suffer a great amount of damage.

In the last decades, new techniques based on adding devices to the buildings with the main objective of dissipating the energy exerted by the earthquake and alleviating the ductility demand on primary structural elements have improved the seismic behavior of the structures [25]. The purpose is to control the seismic response of the buildings by means of a set of dissipating devices. In the case of *passive energy dissipating devices* (EDD) an important part of the energy input is dissipated without the need of an external energy supply.

Several works about seismic control with passive EDDs are available; for example, in reference [4] the response of structures equipped with viscoelastic and viscous devices is compared; in reference [8] an approximated method is used to carry out a comparative study considering metallic and viscous devices. Aiken [1] presents the contribution of the extra energy dissipation due to EDDs as an *equivalent damping* added to the linear bare structure. A critical review of reduction factors and design force levels can be consulted in [10]. A method for the preliminary design of passively controlled buildings is presented in reference [3].

The design methods proposed for RC structures are mainly based on supposing that the behavior of the bare structure remains elastic, while the energy dissipation relies on the control system. However, experimental and theoretical evidence show that inelastic behavior can also occur in the structural elements during severe earthquakes [20]. In order to perform a precise dynamic nonlinear analysis of passively controlled buildings sophisticated numerical tools became are necessary [14] .

Considering that most of the elements in RC buildings are columns and beams, one dimensional formulations for structural elements appear as a solution combining both numerical precision and reasonable computational costs [11]. An additional refinement is obtained considering an arbitrary distribution of materials on the beam cross section [18], and in this case, the constitutive relationship at cross sectional level is deduced by integration. Formulations considering both, constitutive and geometric nonlinearity are rather scarce; most of the geometrically nonlinear models are limited to the elastic case [7, 21] and the inelasticity has been restricted mainly to plasticity [24]. Recently, Mata *et.al.* [11, 12] have extended the geometrically exact formulation for beams due to Reissner-Simo [19, 21, 23] to an arbitrary distribution of composite materials on the cross sections for the static and dynamic cases.

From the numerical point of view, EDDs usually have been described in a global sense by means of force–displacement or moment–curvature relationships [25] which intend to capture appropriately the energy dissipating capacity of the devices [13].

In this work, a fully geometric and constitutive nonlinear formulation for beam elements is developed. A fiber–like approach is used for representing arbitrary distributions of composite materials on the plane beam cross sections. EDDs are considered as beam elements without rotational degrees of freedom. Thermodynamically consistent constitutive laws are provided for steel, concrete and EDDs. The mixing rule is employed for the treatment of the resulting composite. A brief description of the damage indices capable of estimate the remaining load carrying capacity of the buildings is also given. Finally, the numerical simulation of the seismic behavior of a precast RC structure with EDDs is presented.

2 FINITE DEFORMATION FORMULATION FOR STRUCTURAL ELEMENTS

2.1 Beam model

The original geometrically exact formulation for beams due to Simo and Vu Quoc [21, 22] is expanded here for considering an intermediate curved reference configuration according to [7]. Let $\{\hat{E}_i\}$ and $\{\hat{e}_i\}$ be the spatially fixed *material* and *spatial* frames[1], respectively. The straight reference beam is defined by the curve $\hat{\varphi}_{00} = S\hat{E}_1$, with $S \in [0, L]$ its arch–length coordinate. Beam cross sections are described by means of the coordinates ξ_β directed along $\{\hat{E}_\beta\}$. The curved reference beam is defined by means of the spatially fixed curve given by $\hat{\varphi}_0 = \sum_i \varphi_{0i}(S)\hat{e}_i \in \mathbb{R}^3$. Each point on this curve has rigidly attached an orthogonal local frame $\hat{t}_{0i}(S) = \mathbf{\Lambda}_0\hat{E}_i \in \mathbb{R}^3$, where $\mathbf{\Lambda}_0 \in SO(3)$ is the orientation tensor[2]. The planes of the cross sections are normal to the vector tangent to the reference curve[3], *i.e.* $\hat{\varphi}_{0,S} = \hat{t}_{01}(S)$. The position vector of a material point on the curved reference beam is $\hat{x}_0 = \hat{\varphi}_0 + \sum_\beta \mathbf{\Lambda}_0\xi_\beta\hat{E}_{0\beta}$. The motion deforms points on the curved reference beam from $\hat{\varphi}_0(S)$ to $\hat{\varphi}(S,t)$ (at time t) and the local orientation frame is simultaneously rotated together with the beam cross section, from $\mathbf{\Lambda}_0(S)$ to $\mathbf{\Lambda}(S,t)$ by means of the *incremental rotation tensor* as $\mathbf{\Lambda} = \mathbf{\Lambda}_n\mathbf{\Lambda}_0 \in SO(3)$. In general, $\hat{t}_1$ does not coincides with $\hat{\varphi}_{,S}$ because of the shearing [21]. The position vector of a material point on the current beam is

$$\hat{x}(S,\xi_\beta,t) = \hat{\varphi}(S,t) + \sum_\beta \xi_\beta\hat{t}_\beta(S,t) = \hat{\varphi} + \sum_\beta \mathbf{\Lambda}\xi_\beta\hat{E}_\beta \tag{1}$$

Eq. (1) implies that the current beam configuration is determined by $(\hat{\varphi}, \mathbf{\Lambda})$. The deformation gradients of the curved reference beam and of the current beam referred to the straight beam are denoted by $\mathbf{F}_0$ and $\mathbf{F}$, respectively. The deformation gradient $\mathbf{F}_n := \mathbf{F}\mathbf{F}_0^{-1}$ is responsible for the development of strains and can be expressed as [9, 11]

$$\mathbf{F}_n = \mathbf{F}\mathbf{F}_0^{-1} = \frac{1}{|\mathbf{F}_0|}[\hat{\varphi}_{,S} - \hat{t}_1 + \widetilde{\boldsymbol{\omega}}_n \sum_\beta \xi_\beta\hat{t}_\beta] \otimes \hat{t}_{01} + \mathbf{\Lambda}_n \tag{2}$$

where $|\mathbf{F}_0|$ is the determinant of $\mathbf{F}_0$ and $\widetilde{\boldsymbol{\omega}}_n \equiv \mathbf{\Lambda}_{n,S}\mathbf{\Lambda}_n^T$ is the curvature tensor relative to the curved reference beam. In Eq. (2) the term defined as $\hat{\gamma}_n = \hat{\varphi}_{,S} - \hat{t}_1$ corresponds to the reduced strain measure of shearing and elongation [9, 21] with material description given by $\hat{\Gamma} = \mathbf{\Lambda}^T\hat{\gamma}$. The material representation of $\mathbf{F}_n$ is obtained as $\mathbf{F}_n^{\mathrm{m}} = \mathbf{\Lambda}^T\mathbf{F}_n\mathbf{\Lambda}_0$. It is possible to construct the strain tensor $\boldsymbol{\varepsilon}_n = \mathbf{F}_n - \mathbf{\Lambda}_n$, which conjugated to the asymmetric *First Piola Kirchhoff* (FPK) stress tensor $\mathbf{P} = \hat{P}_i \otimes \hat{t}_{0i}$ referred to the curved reference beam [21]. The spatial strain vector acting on the current beam cross section is obtained as $\hat{\varepsilon}_n = \boldsymbol{\varepsilon}_n\hat{t}_{01}$ and the spatial *stress resultant* $\hat{n}$ and *stress couple* $\hat{m}$ vectors can be estimated from $\hat{P}_1$ according to

$$\hat{n}(S) = \int_{\mathcal{A}} \hat{P}_1 d\mathcal{A}; \qquad \hat{m}(S) = \int_{\mathcal{A}} (\hat{x} - \hat{\varphi}) \times \hat{P}_1 d\mathcal{A} \tag{3}$$

The material form of $\hat{P}_j$, $\hat{\varepsilon}_n$, $\hat{n}$ and $\hat{m}$ are obtained as $\hat{\mathcal{E}}_n = \mathbf{\Lambda}^T\hat{\varepsilon}_n$, $\hat{P}_j^{\mathrm{m}} = \mathbf{\Lambda}^T\hat{P}_j$, $\hat{m}^{\mathrm{m}} = \mathbf{\Lambda}^T\hat{n}$ and $\hat{m}^{\mathrm{m}} = \mathbf{\Lambda}^T\hat{m}$, respectively. An objective measure of the *strain rate* vector $\hat{s}_n$ acting on any

[1]The indices i and β range over $\{1, 2, 3\}$ and $\{2, 3\}$, respectively.

[2]The symbol $SO(3)$ is used to denote the finite rotation manifold [21, 22].

[3]The symbol $(\bullet)_{,x}$ is used to denote partial differentiation of $(\bullet)$ with respect to x.

material point can be deduced using the definition of the Lie derivative operator $[\overset{\triangledown}{\bullet}]$ [11, 12] as follows:

$$\hat{s}_n = [\overset{\triangledown}{\dot{\hat{\varepsilon}}_n}] = [\overset{\triangledown}{\dot{\hat{\gamma}}_n}] + [\overset{\triangledown}{\dot{\tilde{\boldsymbol{\omega}}}_n}] \sum_\beta \xi_\beta \hat{t}_\beta = \dot{\hat{\varphi}},_S - \widetilde{\boldsymbol{v}}_n \hat{\varphi},_S + \widetilde{\boldsymbol{v}}_n,_S \sum_\beta \xi_\beta \hat{t}_\beta \tag{4}$$

where $\widetilde{\boldsymbol{v}}_n \equiv \dot{\boldsymbol{\Lambda}}_n \boldsymbol{\Lambda}_n^T$ is the current *spin* or angular velocity of the beam cross section with respect to the curved reference beam. The material form of Eq. (4) is $\hat{\mathcal{S}}_n = \boldsymbol{\Lambda}^T \hat{s}_n$.

The classical form of the *equations of motion of the Cosserat beam* for the static case are

$$\hat{n},_S + \hat{n}_p = \mathcal{A}_{\rho 0} \ddot{\hat{\varphi}} + \underbrace{\widetilde{\boldsymbol{\alpha}}_n \hat{\mathcal{S}}_{\rho 0} + \widetilde{\boldsymbol{v}}_n \widetilde{\boldsymbol{v}}_n \hat{\mathcal{S}}_{\rho 0}}_{D_1} \tag{5a}$$

$$\hat{m},_S + \hat{\varphi},_S \times \hat{n} + \hat{m}_p = \mathcal{I}_{\rho 0} \hat{\alpha}_n + \widetilde{\boldsymbol{v}}_n \mathcal{I}_{\rho 0} \hat{v}_n + \underbrace{\hat{\mathcal{S}}_{\rho 0} \times \ddot{\hat{\varphi}}}_{D_2} \tag{5b}$$

where $\hat{n}_p$ and $\hat{m}_p$ are the external *body force* and *body moment* per unit of reference length at time t, $\mathcal{A}_{\rho 0}$, $\hat{\mathcal{S}}_{\rho 0}$ and $\mathcal{I}_{\rho 0}$ are the cross sectional mass density, the first mass moment density and the second mass moment density per unit of length of the curved reference beam, respectively; their explicit expressions can be consulted in references [9, 22]. $\widetilde{\boldsymbol{\alpha}}_n \equiv \ddot{\boldsymbol{\Lambda}}_n \boldsymbol{\Lambda}_n^T - \widetilde{\boldsymbol{v}}_n^2$ is the angular acceleration of the beam cross section and $\hat{v}_n$ and $\hat{\alpha}_n$ are the axial vectors of $\widetilde{\boldsymbol{v}}_n$ and $\widetilde{\boldsymbol{\alpha}}_n$, respectively. For most of the practical cases, the terms D_1 and D_2 can be neglected.

Considering a kinematically admissible variation[4] $h \equiv (\delta\hat{\varphi}, \delta\hat{\theta})$ of the pair $(\hat{\varphi}, \boldsymbol{\Lambda})$ [22], taking the dot product with Eqs. (5a) and (5b), integrating over the length of the curved reference beam and integrating by parts, we obtain the nonlinear functional $\mathbf{G}(\hat{\varphi}, \boldsymbol{\Lambda}, h)$ corresponding to the *weak form of the balance equations* [7, 22]

$$\begin{aligned} \mathbf{G}(\hat{\varphi}, \boldsymbol{\Lambda}, h) \; = \; & \int_L \Big[(\delta\hat{\varphi},_S - \delta\hat{\theta} \times \hat{\varphi},_S) \cdot \hat{n} + \delta\hat{\theta},_S \cdot \hat{m} \Big] dS \\ + \; & \int_L \Big[\delta\hat{\varphi} \cdot \mathcal{A}_{\rho_0} \ddot{\hat{\varphi}} + \delta\hat{\theta} \cdot (\mathcal{I}_{\rho_0} \hat{\alpha}_n + \hat{\boldsymbol{v}}_n \mathcal{I}_{\rho_0} \hat{v}_n) \Big] \mathrm{d}S \\ - \; & \int_L \Big[\delta\hat{\varphi} \cdot \hat{n}_p + \delta\hat{\theta} \cdot \hat{m}_p \Big] dS - (\delta\hat{\varphi} \cdot \hat{n} + \delta\hat{\theta} \cdot \hat{m}) \Big|_0^L = 0 \end{aligned} \tag{6}$$

The terms $(\delta\hat{\varphi},_S - \delta\hat{\theta} \times \hat{\varphi},_S)$ and $\delta\hat{\theta},_S$ appearing in Eq. (6) correspond to the co–rotated variations of the reduced strain measures $\hat{\gamma}_n$ and $\hat{\omega}_n$ in spatial description.

2.2 Energy dissipating devices

The finite deformation model for EDDs is obtained from the beam model releasing the rotational degrees of freedom and supposing that all the mechanical behavior of the device is described in terms of the evolution of a unique material point in the middle of the resulting bar.

The current position of a point in the EDD bar is obtained from Eq. (1) and considering that $\xi_\beta = 0$ as $\hat{x}(S,t) = \hat{\varphi}(S,t)$. Supposing that the current orientation of the EDD bar of initial length L^* is given by the tensor $\boldsymbol{\Lambda}^*(t)$, ($\boldsymbol{\Lambda}^*,_S = 0$, $\dot{\boldsymbol{\Lambda}}^* \neq 0$), the spatial position of the *dissipative point* in the EDD is obtained as $\hat{\varphi}(L^*/2, t)$ where $L^*/2$ is the arch–length coordinate of the

[4] Supposing that $\boldsymbol{\Lambda}$ is parameterized in terms of the spatial rotation vector and following the results of reference it is possible to show that $\delta\boldsymbol{\Lambda} = \delta\hat{\theta} \times \boldsymbol{\Lambda}$ with $\delta\hat{\theta}$ an admissible variation of the rotation vector.

middle point in the bar element and the axial strain and the axial strain rate in the dissipative point are obtained from Eqs. (2) and (4) as

$$\hat{\Gamma}_1(t) = \left\{(\mathbf{\Lambda}^{*T}\hat{\varphi}_{,S}) \cdot \hat{E}_1\right\}\Big|_{(L^*/2,t)} - 1 \tag{7a}$$

$$\dot{\hat{\Gamma}}_1(t) = \left\{(\mathbf{\Lambda}^{*T}(\dot{\hat{\varphi}}_{,S} - \widetilde{\boldsymbol{v}}_n\hat{\varphi}_{,S})) \cdot \hat{E}_1\right\}\Big|_{(L^*/2,t)} \approx \frac{\mathrm{d}}{\mathrm{d}t}\hat{\Gamma}_1(t)\Big|_{(L^*/2,t)} \tag{7b}$$

Finally, the contribution of the EDD bar to the functional of Eq. (6), written in the material description, is given by

$$\mathbf{G}_{\text{EDD}} = \int_{L^*} n_1^{\text{m}}\delta\hat{\Gamma}_1 dS + \left\{(\mathbf{\Lambda}^{*T}\delta\hat{\varphi})^T[\mathbf{M}]_{\text{d}}(\mathbf{\Lambda}^{*T}\ddot{\hat{\varphi}})\right\}\Big|_{(L^*/2,t)} \tag{8}$$

where it was assumed that $\mathcal{I}_{\rho_0} \approx 0$, *i.e.* the contribution of the EDDs to the rotational mass of the system is negligible and $[\mathbf{M}]_{\text{d}}$ is the EDD's *translational inertia* matrix. The term $\delta\hat{\Gamma}_1 = (\mathbf{\Lambda}^{*T}(\delta\hat{\varphi}_{,S} - \delta\hat{\theta} \times \hat{\varphi}_{,S}))\cdot\hat{E}_1$ corresponds to the material form of the variation of the axial strain in the EDD.

3 CONSTITUTIVE MODELS

In this work, material points on the cross sections are considered as formed by a *composite material* corresponding to a homogeneous mixture of different simple *components*, each of them with its own constitutive law. The resulting behavior is obtained by means of the *mixing theory*. Two kinds of nonlinear constitutive models for simple materials are used: the *damage* and *plasticity* models. The constitutive models are formulated in terms of the material form of the FPK stress and strain vectors, $\hat{P}_1^{\text{m}}$ and $\hat{\mathcal{E}}_n$, respectively [11, 12].

3.1 Degrading materials: damage model

The progress of the damage is based on the evolution of the scalar damage parameter $d \in [0, 1]$ [15]. Starting from an appropriated form of the free energy density and considering the fulfilment of the Clasius–Plank inequality and applying the Coleman's principle [11] the following constitutive relation in material form is obtained:

$$\hat{P}_1^{\text{m}} = (1-d)\boldsymbol{\mathcal{C}}^{\text{me}}\hat{\mathcal{E}}_n = \boldsymbol{\mathcal{C}}^{\text{ms}}\hat{\mathcal{E}}_n = (1-d)\hat{P}_{01}^{\text{m}} \tag{9}$$

where $\boldsymbol{\mathcal{C}}^{\text{me}}$ and $\boldsymbol{\mathcal{C}}^{\text{ms}} = (1-d)\boldsymbol{\mathcal{C}}^{\text{me}}$ is the *secant constitutive tensor*. Eq. (9) shows that the FPK stress vector is obtained from its elastic counterpart by multiplying it by the factor $(1-d)$.

The damage yield criterion $\mathcal{F}$ [2, 5] is defined as a function of the undamaged elastic free energy density and written in terms of the components of the material form of the undamaged principal stresses, $\hat{P}_{p0}^{\text{m}}$, as

$$\mathcal{F} = \mathcal{P} - f_c = [1 + r(n-1)]\sqrt{\sum_{i=1}^{3}(P_{p0i}^{\text{m}})^2} - f_c \leq 0 \tag{10a}$$

where $\mathcal{P}$ is the equivalent stress, r and n are given in function of the tension and compression strengths f_c and f_t and the parts of the free energy density developed when the tension, $(\Psi_t^0)_L$, or compression, $(\Psi_c^0)_L$, limits are reached and are defined as

$$(\Psi_{t,c}^0)_L = \sum_{i=1}^{3} \frac{\langle \pm P_{p0i}^{\text{m}} \rangle \mathcal{E}_{ni}}{2\rho_0}, \qquad \Psi_L^0 = (\Psi_t^0)_L + (\Psi_c^0)_L \tag{10b}$$

$$f_t = (2\rho \Psi_t^0 E_0)_L^{\frac{1}{2}}, \quad f_c = (2\rho \Psi_c^0 E_0)_L^{\frac{1}{2}}, \quad n = \frac{f_c}{f_t}, \quad r = \frac{\sum_{i=1}^{3} \langle P_{p0i}^{\text{m}} \rangle}{\sum_{i=1}^{3} |P_{p0i}^{\text{m}}|} \tag{10c}$$

A more general expression equivalent to that given in Eq. (10a) [2] is given by

$$\bar{\mathcal{F}} = \mathcal{G}(\mathcal{P}) - \mathcal{G}(f_c), \qquad \mathcal{G}(\chi) = 1 - \frac{\bar{\mathcal{G}}(\chi)}{\chi} = 1 - \frac{\chi^*}{\chi} e^{\kappa(1 - \frac{\chi}{\chi}^*)} \tag{11}$$

where the term $\bar{\mathcal{G}}(\chi)$ gives the initial yield stress for certain value of the scalar parameter $\chi = \chi^*$ and for $\chi \to \infty$ the final strength is zero. The parameter κ is calibrated to obtain an amount of dissipated energy equal to the specific fracture energy of the material $g_f^d = G_f^d / l_c$; where G_f^d is the tensile fracture energy and l_c is the characteristic length of the fractured domain. The evolution law for the internal damage variable d is given by

$$\dot{d} = \dot{\mu} \frac{\partial \bar{\mathcal{F}}}{\partial \mathcal{P}} = \dot{\mu} \frac{\partial \mathcal{G}}{\partial \mathcal{P}} \tag{12}$$

where $\dot{\mu} = \dot{\mathcal{P}} \geq 0$ is the *damage consistency* parameter [11]. Finally, the Kuhn-Thucker relations: (a) $\dot{\mu} \geq 0$, (b) $\bar{\mathcal{F}} \leq 0$, (c) $\dot{\mu}\bar{\mathcal{F}} = 0$, have to be employed to derive the unloading–reloading conditions *i.e.* if $\bar{\mathcal{F}} < 0$ the condition (c) imposes $\dot{\mu} = 0$; on the contrary, if $\dot{\mu} > 0$ then $\mathcal{F} = 0$.

3.1.1 Viscosity

The rate dependent behavior is considered by using the Maxwell model. The FPK stress vector $\hat{P}_1^{\text{mt}}$ is obtained as the sum of a rate independent part $\hat{P}_1^{\text{m}}$, Eq. (9), and a viscous component $\hat{P}_1^{\text{mv}}$ as

$$\hat{P}_1^{\text{mt}} = \hat{P}_1^{\text{m}} + \hat{P}_1^{\text{mv}} = \boldsymbol{\mathcal{C}}^{\text{mv}} \hat{\mathcal{E}}_n + \boldsymbol{\eta}^{sm} \hat{\mathcal{S}}_n = (1-d) \boldsymbol{\mathcal{C}}^{\text{me}} \big(\hat{\mathcal{E}}_n + \frac{\eta}{E_0} \hat{\mathcal{S}}_n \big) \tag{13}$$

where $\boldsymbol{\eta}^{sm} = \eta / E_0 \boldsymbol{\mathcal{C}}^{\text{ms}}$ is the *secant viscous* constitutive tensor, $\boldsymbol{\mathcal{C}}^{\text{mv}} = (1-d)\boldsymbol{\mathcal{C}}^{\text{me}}$, and the parameter η is the viscosity. The linearized increment of the FPK stress vector (material and co–rotated forms) are calculated as

$$\Delta \hat{P}_1^{\text{mt}} = \boldsymbol{\mathcal{C}}^{\text{mv}} \Delta \hat{\mathcal{E}}_n + \boldsymbol{\eta}^{sm} \Delta \hat{\mathcal{S}}_n, \quad \Delta \overset{\triangledown}{[\hat{P}_1^{\text{t}}]} = \boldsymbol{\mathcal{C}}^{\text{sv}} \Delta \overset{\triangledown}{[\hat{\varepsilon}_n]} + \boldsymbol{\eta}^{\text{ss}} \Delta \overset{\triangledown}{[\hat{s}_n]} \tag{14}$$

where $\boldsymbol{\mathcal{C}}^{\text{sv}} = \boldsymbol{\Lambda} \boldsymbol{\mathcal{C}}^{\text{mv}} \boldsymbol{\Lambda}^T$ and $\boldsymbol{\eta}^{\text{ss}} = \boldsymbol{\Lambda} \boldsymbol{\eta}^{\text{sm}} \boldsymbol{\Lambda}^T$. The explicit form of the terms $\Delta \hat{\mathcal{S}}_n$ and $\Delta \overset{\triangledown}{[\hat{s}_n]}$ and he material description of the tangent constitutive tensor $\boldsymbol{\mathcal{C}}^{\text{mv}}$ can be consulted in reference [12].

3.2 Plastic materials

In the case of materials which can undergo non–reversible deformations the plasticity model formulated in the material configuration is used for predicting their mechanical response. Assuming small elastic, finite plastic deformations, an appropriated form of the free energy density and analogous procedures as those for the damage model we have

$$\hat{P}_1^{\text{m}} = \rho_0 \frac{\partial \Psi(\hat{\mathcal{E}}_n^e, k_p)}{\partial \hat{\mathcal{E}}_n^e} = \boldsymbol{\mathcal{C}}^{\text{ms}} \big(\hat{\mathcal{E}}_n - \hat{\mathcal{E}}_n^P \big) = \boldsymbol{\mathcal{C}}^{\text{me}} \hat{\mathcal{E}}_n^e \tag{15}$$

where the $\hat{\mathcal{E}}_n^e$ is the elastic strain calculated subtracting the plastic strain $\hat{\mathcal{E}}_n^P$ from the total strain $\hat{\mathcal{E}}_n$ and ρ_0 is the density in the material configuration.

Both, the yield function, $\mathcal{F}_p$, and plastic potential function, $\mathcal{G}_p$ are formulated in terms of the FPK stress vector $\hat{P}_1^{\mathrm{m}}$ and the plastic damage internal variable k_p as

$$\mathcal{F}_p(\hat{P}_1^{\mathrm{m}}, k_p) = \mathcal{P}_p(\hat{P}_1^{\mathrm{m}}) - f_p(\hat{P}_1^{\mathrm{m}}, k_p) = 0, \quad \mathcal{G}_p(\hat{P}_1^{\mathrm{m}}, k_p) = \mathcal{K} \tag{16}$$

where $\mathcal{P}_p(\hat{P}_1^{\mathrm{m}})$ is the equivalent stress, which is compared with the *hardening* function $f_p(\hat{P}_1^{\mathrm{m}}, k_p)$ and $\mathcal{K}$ is a constant value [16]. In this work, k_p constitutes a measure of the energy dissipated during the plastic process and it is defined [17] as

$$g_f^P = \frac{G_f^P}{l_c} = \int_{t=0}^{\infty} \hat{P}_1^{\mathrm{m}} \cdot \dot{\mathcal{E}}_n^P \mathrm{d}t, \quad 0 \le \left[k_p = \frac{1}{g_f^P} \int_{t=0}^{t} \hat{P}_1^{\mathrm{m}} \cdot \dot{\mathcal{E}}_n^P dt\right] \le 1 \tag{17}$$

where G_f^P is the specific plastic fracture energy of the material in tension and l_c is the length of the fractured domain defined in analogous manner as for the damage model. The integral term in Eq. (17) corresponds to the energy dissipated by means of plastic work.

The flow rules for the internal variables $\hat{\mathcal{E}}_n^P$ and k_p are defined as

$$\dot{\hat{\mathcal{E}}}_n^P = \dot{\lambda} \frac{\partial \mathcal{G}_p}{\partial \hat{P}_1^{\mathrm{m}}}, \quad \dot{k}_p = \dot{\lambda}\, \hat{\varrho}(\hat{P}_1^{\mathrm{m}}, k_p, G_f^P) \cdot \frac{\partial \mathcal{G}_p}{\partial \hat{P}_1^{\mathrm{m}}} = \hat{\varrho}(\hat{P}_1^{\mathrm{m}}, k_p, G_f^P) \cdot \dot{\hat{\mathcal{E}}}_n^P \tag{18}$$

where $\dot{\lambda}$ is the plastic consistency parameter and $\hat{\varrho}$ is the following *hardening* vector [16]. In what regards the hardening function of Eq. (16), the following evolution equation has been proposed:

$$f_p(\hat{P}_1^{\mathrm{m}}, k_p) = r\sigma_t(k_p) + (1-r)\sigma_c(k_p) \tag{19}$$

where r has been defined in Eq. (10c) and the scalar functions $\sigma_t(k_p)$ and $\sigma_c(k_p)$ describe the evolution of the yielding threshold in uniaxial tension and compression tests, respectively.

As it is a standard practice in plasticity, the loading/unloading conditions are derived in the standard form from the Kuhn-Tucker relations formulated for problems with unilateral restrictions, *i.e.* , (a) $\dot{\lambda} \ge 0$, (b) $\mathcal{F}_p \le 0$ and (c) $\dot{\lambda}\mathcal{F}_p = 0$. Explicit expressions of $\dot{\lambda}$ and of the material form of the tangent constitutive tensor can be reviewed in references [11, 16, 17].

3.3 Mixing theory for composites

Each material point on the beam cross section is treated as a composite material according to the *mixing theory* [16]. Supposing N different components coexisting in a generic material point subjected to the same material strain $\hat{\mathcal{E}}_n$, we have the following closing equation: $\hat{\mathcal{E}}_n \equiv (\hat{\mathcal{E}}_n)_1 = \cdots = (\hat{\mathcal{E}}_n)_q = \cdots = (\hat{\mathcal{E}}_n)_N$, which imposes the strain compatibility between components. The free energy density of the composite, $\bar{\Psi}$, is obtained as the weighted sum of the free energy densities of the N components. The weighting factors correspond to the quotient between the volume of the q^{th} component, V_q, and the total volume, V, such that $\sum_q k_q = 1$. The material form of the FPK stress vector $\hat{P}_1^{\mathrm{mt}}$ for the composite, including the participation of rate dependent effects, is obtained in analogous way as for simple materials *i.e.*

$$\hat{P}_1^{\mathrm{mt}} \equiv \sum_q^N k_q (\hat{P}_1^{\mathrm{m}} + \hat{P}_1^{\mathrm{mv}})_q = \sum_q^N k_q \left[(1-d)\boldsymbol{\mathcal{C}}^{\mathrm{me}}\left(\hat{\mathcal{E}}_n + \frac{\eta}{E_0}\hat{\mathcal{S}}_n\right)\right]_q \tag{20}$$

where $(\hat{P}_1^{\mathrm{m}})_q$ and $(\hat{P}_1^{\mathrm{mv}})_q$ correspond the strain and rate dependent stresses of each one of the N components. The material form of the secant and tangent constitutive tensors for the composite, $\bar{\boldsymbol{\mathcal{C}}}^{\mathrm{ms}}$ and $\bar{\boldsymbol{\mathcal{C}}}^{\mathrm{mt}}$, are obtained in an analogous manner [16].

3.4 Constitutive relations for EDDs

The constitutive law proposed for EDDs is based on a previous work of the authors [13] which provides a versatile strain–stress relationship with the following general form:

$$\bar{P}^{\mathrm{m}}(\mathcal{E}_1, \dot{\mathcal{E}}_1, t) = \bar{P}_1^{\mathrm{m}}(\mathcal{E}_1, t) + \bar{P}_2^{\mathrm{m}}(\dot{\mathcal{E}}_1, t) \tag{21}$$

where $\bar{P}^{\mathrm{m}}$ is the average stress in the EDD, $\mathcal{E}_1$ the strain level, t the time, $\dot{\mathcal{E}}_1$ the strain rate, $\bar{P}_1^{\mathrm{m}}$ and $\bar{P}_1^{\mathrm{m}}$ are the strain dependent and rate dependent parts of the stress, respectively. The model uncouples the total stress in viscous and non-viscous components, which correspond to a viscous dashpot device acting in parallel with a nonlinear hysteretic spring. The purely viscous component does not requires to be a linear function of the strain rate. Additionally, hardening, and variable elastic modulus can be reproduced [13].

4 NUMERICAL IMPLEMENTATION

In order to obtain a numerical solution, the linearized form of Eq. (6) is written as

$$\boldsymbol{\mathcal{L}}[\mathbf{G}(\hat{\varphi}_*, \boldsymbol{\Lambda}_*, h)] = \mathbf{G}(\hat{\varphi}_*, \boldsymbol{\Lambda}_*, h) + D\mathbf{G}(\hat{\varphi}_*, \boldsymbol{\Lambda}_*, h) \cdot p \tag{22}$$

where $\boldsymbol{\mathcal{L}}[\mathbf{G}(\hat{\varphi}_*, \boldsymbol{\Lambda}_*, h)]$ is the linear part of the functional $\mathbf{G}(\hat{\varphi}, \boldsymbol{\Lambda}, h)$ at the configuration $(\hat{\varphi}, \boldsymbol{\Lambda}) = (\hat{\varphi}_*, \boldsymbol{\Lambda}_*)$ and $p \equiv (\Delta\hat{\varphi}, \Delta\hat{\theta})$ is an admissible variation. The term $\mathbf{G}(\hat{\varphi}_*, \boldsymbol{\Lambda}_*, h)$ supplies the *unbalanced force* and it is composed by the contributions of the inertial, external and internal terms; and $D\mathbf{G}(\hat{\varphi}_*, \boldsymbol{\Lambda}_*, h) \cdot p$, gives the *tangential stiffness* [22].

The linearization of the inertial and external components, $D\mathbf{G}_{\text{ine}} \cdot p$ and $D\mathbf{G}_{\text{ext}} \cdot p$ give the inertial and load dependent parts of the tangential stiffness, $\mathbf{K}_{\mathrm{I}*}$ and $\mathbf{K}_{\mathrm{P}*}$, respectively, and it can be consulted in [22, 23]. The linearization of the internal term is calculated as

$$D\mathbf{G}_{\text{int}} \cdot p = \underbrace{\int_{[0,L]} h^T [\mathbf{B}_*]^T [\mathbf{C}_*^{\text{st}}][\mathbf{B}_*] p \, dS}_{\mathbf{K}_{\mathrm{M}*}} + \underbrace{\int_L h^T \cdot [\mathbf{B}_*]^T [\boldsymbol{\Upsilon}_*^{\text{st}}][\mathcal{V}_*] p \, dS}_{\mathbf{K}_{\mathrm{V}*}} \tag{23}$$

$$+ \underbrace{\int_{[0,L]} h^T ([\widetilde{\boldsymbol{n}}_{S*}] - [\mathbf{B}_*]^T [\widetilde{\mathbf{F}}_*]) p \, dS}_{\mathbf{K}_{\mathrm{G}*}} \tag{24}$$

where the operators $[\mathbf{C}_*^{\text{st}}]$, $[\boldsymbol{n}_{S*}]$, $[\mathbf{B}_*]$, $[\boldsymbol{\Upsilon}_*^{\text{st}}]$, $[\mathcal{V}_*]$ and $[\widetilde{\mathbf{F}}_*]$ can be consulted in references [9, 11, 12, 22]. The linearized terms $\mathbf{K}_{\mathrm{G}*}$, $\mathbf{K}_{M*}$ and $\mathbf{K}_{\mathrm{V}*}$, evaluated at the configuration $(\hat{\varphi}_*, \boldsymbol{\Lambda}_*)$, give the *geometric*, *material* and *viscous* parts of the tangent stiffness, which allows to rewrite Eq. (22) as

$$\boldsymbol{\mathcal{L}}[\mathbf{G}_*] = \mathbf{G}_* + \mathbf{K}_{\mathrm{I}*} + \mathbf{K}_{\mathrm{M}*} + \mathbf{K}_{\mathrm{V}*} + \mathbf{K}_{\mathrm{G}*} + \mathbf{K}_{\mathrm{P}*} \tag{25}$$

The solution of the discrete form of Eq. (25) by using the FE method follows identical procedures as those described in [22] for an iterative Newton-Rapson integration scheme and it will not be included here.

5 DAMAGE INDICES

A measure of the damage level of a material point can be obtained as the ratio of the existing stress level to its elastic counter part. Following this idea, it is possible to define the fictitious damage variable $\check{D}$ as [2]

$$\sum_{i=1}^{3} |P_{1i}^{\mathrm{m}}| = (1 - \check{D}) \sum_{i=1}^{3} |P_{1i0}^{\mathrm{m}}| \rightarrow \check{D} = 1 - \frac{\sum_{i=1}^{3} |P_{1i}^{\mathrm{m}}|}{\sum_{i=1}^{3} |P_{1i0}^{\mathrm{m}}|} \tag{26}$$

where $|P_{1i}^{\mathrm{m}}|$ and $|P_{1i0}^{\mathrm{m}}|$ are the absolute values of the components of the existing and elastic stress vectors, respectively. Initially, the material remains elastic and $\check{D} = 0$, but when all the energy of the material has been dissipated $|P_{1i}^{\mathrm{m}}| \rightarrow 0$ and $\check{D} \rightarrow 1$. Eq. (26) can be extended to consider elements or even the whole structure by means of integrating over a finite volume as explained in reference [11].

6 NUMERICAL EXAMPLES

6.1 Seismic response of a precast RC building with EDDs

The nonlinear seismic response of a typical precast RC industrial building shown in Figure 1 is studied. The building has a bay width of 24 m and 12 m of inter–axes length. The story hight is 12 m. The compression limit of the concrete is 35 MPa with an elastic modulus of 290.000 MPa. It has been assumed a Poisson coefficient of 0.2. The ultimate tensile stress for the steel is 510 MPa. This figure also shows some details of the steel reinforcement of the cross sections. The dimensions of the columns are 60x60 cm^2. The beam has an initial high of 60 cm on the supports and 160 cm in the middle of the span. The permanent loads considered are 1050 N/m^2 and the weight of upper half of the closing walls with 432,000 N. The input acceleration corresponds to the N–S component of the EL Centro 1940 earthquake.

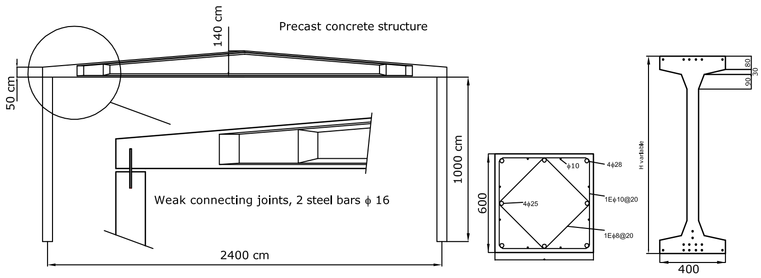

Figure 1: Description of the structure.

The half part of the building is meshed using 4 quadratic elements with two Gauss integration points for the resulting beam and column. The EED element was calibrated for reproducing a plastic dissipative mechanism. The properties of the device were: a yielding force of 150.000 N for a displacement of 1.5 mm.

The results of the numerical simulations allow seeing that the employment of plastic EDDs contributes to improve the seismic behavior of the structure. Figure 2a shows the hysteretic cycles obtained from the lateral displacement of the upper beam–column joint and the horizontal reaction (base shear) in the columns for the structure with and without devices. It is possible to appreciate that the non–controlled structure (bare frame) presents greater lateral displacements and more structural damage. Figure 2b shows the hysteretic cycles obtained in the EDD, evidencing that part of the dissipated energy is concentrated in the controlling devices, as expected.

Figure 3 shows the time history response of the horizontal displacement of the upper beam–column joint. A reduction of approximately 40 % is obtained for the maximum lateral displacement when compared with the bare frame. A possible explanation for the limited effectiveness

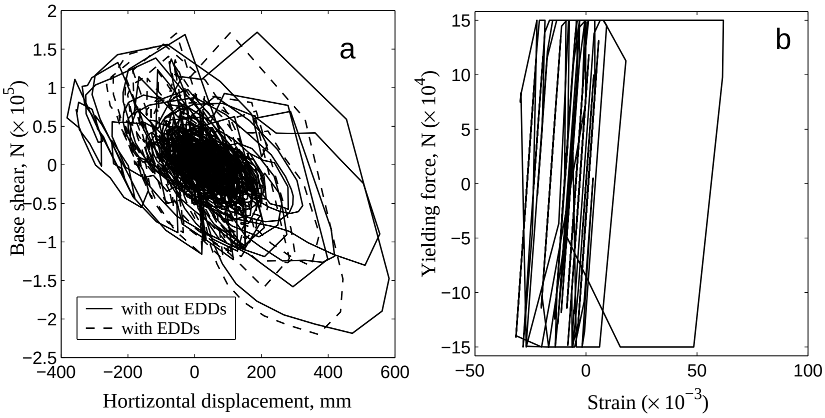

Figure 2: a: base shear–displacement relationship. b: Hysteretic cycles in the EDD.

of the EDD is that the devices only contribute to increase the ductility of the beam–column joint without alleviating the base shear demand on the columns due to the dimensions of the device and its location in the structure.

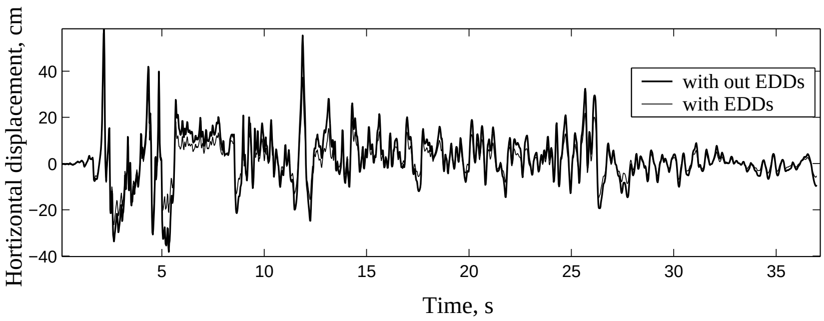

Figure 3: Time history of the top horizontal displacement.

7 CONCLUSIONS

In this work, a geometrically exact formulation for initially curved beams has been extended to consider arbitrary distributions of composite materials on the cross sections in the seismic case. The consistent linearization of the weak form of the momentum balance equations considers the constitutive nonlinearity with rate dependent effects. The resulting model is implemented in a displacement based FEM code. An iterative Newton-Rapson scheme is used for the solution of the discrete version of the linearized problem. An specific element for EDD is developed, based on the beam model but releasing the rotational degrees of freedom. Each material point of the cross section is assumed to be composed of several simple materials with their own constitutive laws. The mixing rule is used to describe the resulting composite. Viscosity is included at constitutive level by means of a Maxwell model. Beam cross sections are meshed into a grid of quadrilaterals corresponding to fibers directed along the beam axis. Two additional integration loops are required at cross sectional level in each integration point to obtain the reduced quantities. Local and global damage indices have been developed based on the ratio between the visco elastic and nonlinear stresses. The present formulation is validated by means of a numerical example: the study of the seismic response of a RC precast structure with EDDs.

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REFERENCES

[1] Ian Aiken. Passive energy dissipation hardware and applications. *Proceedings, Lo Angeles County and Seaosc Symposium on Passive Energy Dissipation Systems for New and Existing Buildings*, Los Angeles, July 1996;

[2] A.H. Barbat, S. Oller, A. Hanganu, E. Oñate. Viscous damage model for Timoshenko beam structures. *International Journal for Solids and Structures*, 1997; **34**(30):3953–3976.

[3] J.J. Connor, A. Wada, M. Iwata, Y.H. Huang. Damage-controlled structures. I: Preliminary design methodology for seismically active regions. *Journal of Structural Engineering*, 1997; **123**(4):423–431.

[4] Yaomin Fu, Kazuhiko Kasai. Comparative study of frames using viscoelastic and viscous dampers. *Journal fo Structural Engineering*, 1998; **124**(5):513–522.

[5] A.D. Hanganu, E. Oñate, A.H. Barbat. Finite element methodology for local/global damage evaluation in civil engineering structures. *Conputers and Structures*, 2002; **80**:1667–1687.

[6] Robert D. Hanson, Ian D. Aiken, Douglas K. Nims, Philip J. Ritchter, Robert E. Batchman. State of the art and state of the practice in seismic engineering dissipation. *Procedings, ATC-17-1. Seminar on seismic isolation, passive energy dissipation and active control. Applied Technology Council, San Francisco, California*, March 1993;

[7] A. Ibrahimbegović. On finite element implementation of geometrically nonlinear Reissners beam theory: three-dimensional curved beam elements. *Computer Methods in Applied Mechanics and Engineering*, 1995; **122**:11–26.

[8] Kasuhiko Kasai, Yaomin Fu, Atsushi Watanabe. Passive control systems for seismic damage mitigation. *Journal of Structural Engineering*, 1998; **124**:(5):501–512.

[9] R.K. Kapania, J. Li. On a geometrically exact curved/twisted beam theory under rigid cross-section assumption. *Computational Mechanics*, 2003; **30**:428–443.

[10] Y.Y. Lin, K.C. Chang. Study on damping reduction factors for buildings under earthquake ground motions. *Journal of Structural Engineering, JEE*, 2003; **129**(2):206–214.

[11] P. Mata A, S. Oller, A.H. Barbat. Static analysis of beam structures under nonlinear geometric and constitutive behavior. *Computer Methods in Applied Mechanic and Engineering*, 2006; (Submited).

[12] P. Mata A, S. Oller, A.H. Barbat. Dynamic analysis of beam structures considering geometric and constitutive nonlinearity. *Computer Methods in Applied Mechanic and Engineering*, 2006; (Submited).

[13] Pablo Mata A, Rubén Boroschek K, Alex H. Barbat, Sergio Oller. High damping rubber model for energy dissipating devices. *Journal of Earthquake Engineering, JEE*, 2006; (Accepted).

[14] P. Mata A, S. Oller, A.H. Barbat, R. Boroschek. Numerical code for seismic analysis of structures incorporating energy dissipating devices. *First European Conference on Earthquake Engineering and Seismology, ECEES*, Geneva, Switzerland, 3-8 September 2006.

[15] Oliver J., Cervera M., Oller S., Lubliner J. Isotropic damage models and smeared crack analysis of concrete. *Proceedings 2nd ICCAADCS, Zell Am See, Austria, Pineridge Press*, 1990; **2**:945–958.

[16] S. Oller, E. Oñate, J. Miquel, S. Botello. A plastic damage constitutive model for composites materials. *International Journal of Solids and Structures*, 1996; **33**(17):2501–2518.

[17] S. Oller, E. Oñate, J. Miquel, Mixing anisotropic formulation for the analysis of composites, Communications in Numerical Methods in Engineering. 12 (1996) 471–482.

[18] Phani Kumar V.V. Nukala, Donal W. White. A mixed finite element for three-dimensional nonlinear analysis of steel frames. *Computer Methods in Applied Mechanic and Engineering*, 2004; **193**(5):2507–2545.

[19] E. Reissner. On one-dimensional large-displacement finite-strain beam theory. *Studies in Applied Mathematics*, 1973; LII, **2**87–95.

[20] K.L. Shen, T.T Soong. Design of energy dissipation devices based on concept of damage control. *Journal of Structural Engineering*, 2005; **122**(1):76–82.

[21] J.C. Simo. A finite strain beam formulation. The three-dimensional dynamic problem. Part I. *Computer Methods in Applied Mechanic and Engineering*, 1985; **49**:55–70.

[22] J.C. Simo, L. Vu-Quoc. A three-dimensional finite-strain rod model. Part II: Computational aspects. *Computer Methods in Applied Mechanics and Engineering*, 1986; **58**:79–116.

[23] J.C. Simo, L. Vu-Quoc. On the dynamics in space of rods undergoing large motions - A geometrically exact approach. *Computer Methods in Applied Mechanics and Engineering*, 1988; **66**:125–161.

[24] J.C. Simo, Keith D. Hjelmstad, Robert L. Taylor. Numerical formulations of elasto-viscoplastic response of beams accounting for the effect of shear. *Computer Methods in Applied Mechanics and Engineering*, 1984; **42**:301–330.

[25] T.T. Soong, G.F. Dargush. *Passive Energy Dissipation Systems in Structural Engineering*, 1997; John Wiley & Soms Ltda.